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Performance and Optimization of Induced Strain Actuated Structures Under External Loading

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This paper addresses the actuator/substrate thickness ratio that maximizes the static/dynamic response of a beam for a variety of actuator-structure integration configurations, actuator activation levels, and inertia/external loading conditions. The three different actuator-structure integration configurations considered are actuators embedded inside the substrate, actuators bonded to the surface of the substrate, and actuators discretely attached and offset from the surface of the substrate. In the category of embedded actuators, a shallow embedment just beneath the surface usually maximizes the moment/induced curvature, but for stiffer actuators, a deeper embedment results in a greater induced curvature. For surface-bonded actuators, a physical basis for the existence of an optimum thickness ratio is presented. For the discretely attached actuator configuration, the induced curvature can be more than doubled depending on the actuator/substrate stiffness ratio. The optimum thickness ratio besides the local actuator/beam properties is shown to be a function of the external loads, boundary conditions, and actuator activation level. The effect of thickness ratio on the dynamic response is also investigated. Anomalies in beam response using some of the prevalent methods for computing the static and dynamic responses of such systems are discussed.

Nomenclature

b	= beam width
d	= actuator offset distance
$\bar{d}$	= normalized actuator offset distance, d/t_b
E	= Young's modulus
I	= moment of inertia of beam cross section
M_Λ	= thermal moment applied to the beam due to the actuation of actuator pair
T	= beam/actuator thickness ratio, (t_b/t_a)
t	= thickness
w	= beam transverse displacement
κ	= beam curvature
Λ	= actuator free induced strain
σ_{blocking}	= axial stress in the actuator when it is fully blocked
ψ	= beam-actuator stiffness ratio, $E_b t_b / E_a t_a$
$(\,)_{,x}$	= derivative with respect to coordinate x

Subscripts

a	= actuator properties
b	= beam properties

Introduction

THE design objective in an adaptive structure is to achieve the desired level of noise, vibration, or shape control with the minimum added mass and energy input into the system. To meet this aim, optimization can be performed on two levels. On one level the best location of the actuators is determined; on another level the best actuator properties such as thickness and Young's modulus are determined. Within the second level, again the parameter to be maximized could be the amount of strain energy transferred from the actuator to the substrate or it could be simply the amount of strain induced in the substrate. In this paper the parameters that result in the maximum induced strain in the substrate in flexure are investigated.

Research in the area of optimization of actuator properties is limited, perhaps simply because researchers have chosen to work with whatever actuator materials are available from manufacturers. But it is important to know the optimum values of the parameters involved so that one can be aware of what is achievable and also of the cost of deviating from the optimum parameters.

Optimization for maximizing the strain induced in the substrate was first addressed by Anderson and Crawley[1] and has more recently been addressed by Kim and Jones.[2] Anderson and Crawley considered both embedded and surface-bonded actuators and concluded that actuators embedded just below the surface resulted in the maximum amount of induced strain in the substrate. This observation is not true in general, and it will be shown that for stiffer actuators it is more beneficial to embed deeper. Kim and Jones, extending the models of Anderson and Crawley and Dimitriadis et al.,[3] took a generalized approach to determining if an optimal design of piezoelectric actuators exists. They specifically considered two kinds of substrates, i.e., steel and aluminum, and showed that there is an optimal thickness of surface-bonded piezoelectric actuators that approaches half of the plate thickness for steel and a quarter of the plate thickness for aluminum. They did not consider the effect of external loads on the optimum parameters, and as will be shown, external loads can result in a substantial change in the optimum parameters.

In this paper we consider all three configurations of integrating actuators into structures, i.e., embedding the actuator into the substrate, bonding the actuator to the surface of the structure, and offsetting the actuator from the surface, and ask the following questions: in a given situation 1) which configuration is best and 2) what are the best parameters for this configuration? To answer these questions we take a slightly different approach in that we view the actuator as a natural material system that develops and exerts a force depending on the structural impedance it works against. Viewing the actuator in this manner provides easy answers to many optimization questions. Before further discussion of the behavior of the actuator and how it relates to optimization, it is instructive to study the force-displacement plot of an induced strain actuator, as shown in Fig. 1. When the actuator is fully constrained it develops maximum force (known as the blocking force), as indicated by the point A; when there is no constraint, the actuator expands/contracts freely to its limit (point B, i.e., free induced strain Λ), and no force is developed. When the actuator is coupled to a

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system or structure, depending on the actuator/structure stiffness ratio, the actuator develops a certain force and at the same time undergoes a certain stroke and an equilibrium state that lies on the line AB is established. In most structural control applications it is desirable to maximize the moment applied to the structure, and this does not necessarily correspond to the highest developed actuator force. By moving an actuator away from the neutral axis of the structure, the resistance offered to the expansion/contraction of the actuator is decreased, which results in the development of a smaller force in the actuator. Does a smaller force also mean a smaller moment? The answer is no. The moment applied to the structure, which is a product of the developed actuator force and the distance from the neutral axis, can increase even though the force in the actuator is decreasing. In fact, for all actuator/substrate configurations, for a given thickness and modulus ratio there is an optimum offset distance that maximizes the moment. This does not apply to surface-bonded actuators where the option to vary the offset distance independent of the basic structural flexural stiffness is not available.

First we start by examining actuator systems under no external load other than that due to the constraint of the substrate immediately below the actuator. In the second part of the paper, we study the effect of external loads on the performance and optimum parameters.

Optimization of Actuator-Substrate Thickness Ratio with No External Loads

Embedded Actuators

An expression for the curvature induced due to symmetrically embedded actuators activated out of phase can be developed (based on the Bernoulli-Euler assumptions) using the following equation[4]:

$$M_\Lambda = (EI)_{\text{beam + actuator}}\kappa \tag{1}$$

where

$$M_\Lambda = E_a b d t_a \Lambda \tag{2}$$

and

$$(EI)_{\text{beam + actuator}} = E_b t_b^3\left[\frac{1}{12} - \frac{2}{T}\left(\frac{1}{T^2} + \frac{1}{4}\bar{d}^2\right)\right] \tag{3}$$

Substituting M_Λ and $(EI)_{\text{beam+actuator}}$ from Eqs. (2) and (3) into Eq. (1), we obtained the following expression for the normalized beam surface strain:

$$\frac{\kappa t_b}{\Lambda} = \frac{\bar{d}/T}{\frac{E_b}{E_a}\left[\frac{1}{12} - \frac{2}{T}\left(\frac{1}{T^2} + \frac{\bar{d}^2}{4}\right)\right] + \frac{1}{6T^3} + \frac{\bar{d}^2}{2T}} \tag{4}$$

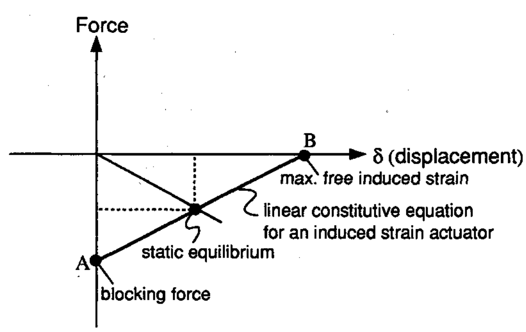

Fig. 1 Force displacement plot of an induced strain actuator indicating the static equilibrium state when the actuator is attached to a substrate.

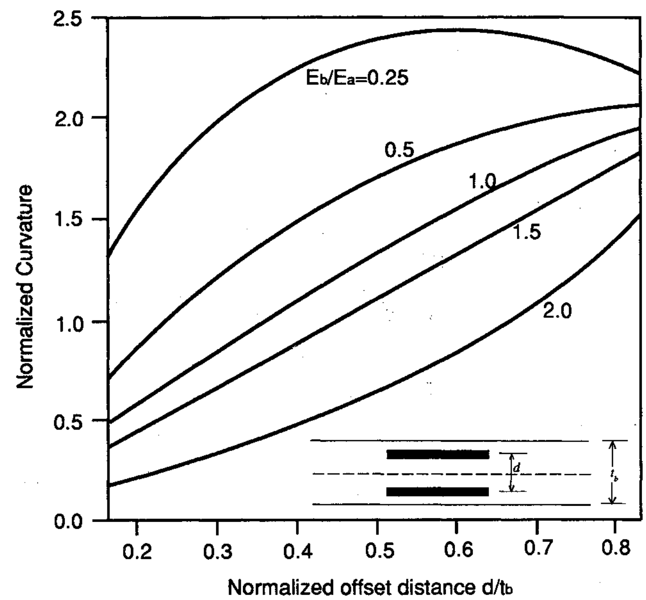

Fig. 2 Beam surface strain/free induced strain vs the normalized offset distance.

Figure 2 shows a plot for the normalized beam surface strain vs the normalized offset distance $\bar{d}$ for a thickness ratio of 6 and different values of the beam-actuator modulus ratios. Although for most practical modulus ratios the optimum distance is approximately 0.9, i.e., when the actuator is embedded just below the surface and flush with it,[1] for $E_b/E_a = 0.25$ the offset distance that maximizes the strain induced in the substrate is about 30% of the beam thickness. Also the strain induced in the substrate is more than for all other modulus ratios. Although this finding has no practical relevance at the present time, it indicates that it will be better to embed actuators deeper below the surface with development of new stiffer actuators [hard PZTs (piezoelectrics)]. It is interesting to note that, for most practical thickness and modulus ratios, actuators embedded just below the surface and flush with it induce the greatest amount of strain in the substrate, much more than is possible with surface-bonded actuators. This is because when the actuator is embedded within the substrate it replaces some of the structural material, whereas when it is surface bonded it adds to the structural stiffness over and above that of the basic structure.

Surface-Bonded Actuators

The preceding discussion applies to materials such as composites where it is possible to embed the actuator within the structure, but for most structures the actuators are bonded to the surface. In such a scenario, the offset distance is fixed and determined by the thickness of the substrate. The only parameter that can then be optimized is the actuator thickness relative to the substrate. Optimization for maximizing induced curvature leads to a thickness ratio of 2–3,[1,2] whereas maximizing strain energy transfer leads to a thickness ratio of 5–10.[1] However we would like to present a physical basis for the existence of an optimum thickness ratio. By optimum here we mean the thickness ratio that results in maximum curvature or beam surface strain. The physical basis lies in studying the effect of the thickness ratio on the through-the-thickness normal stress variation. Figure 3 shows the through-the-thickness stress distribution for three different thickness ratios (for $E_b/E_a = 1$). For the thickness ratio $t_b/t_a = 2.75$, which results in maximum curvature/surface strain, the stress on the top surface of the actuator goes to zero, and this condition in fact determines the optimum thickness ratio. At suboptimum thickness ratios such as $t_b/t_a = 2$, part of the actuator surface goes into tension as seen in Fig. 3a. And in such a situation, the top and bottom actuators not only bend the substrate but also begin to work against each other, and this is what causes the curvature and induced beam surface strain to drop.

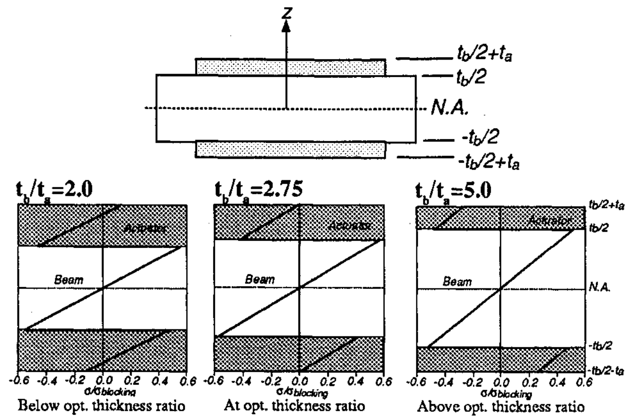

Fig. 3 Through-the-thickness normal stress distribution for a patch of symmetric actuators.

Offset Actuators

The induced strain/curvature for surface-bonded actuators can be increased only in one way and that is by discretely attaching and offsetting the actuators from the beam/plate surface. If the space between the actuator and the beam surface is filled with a honeycomb-type filler that has negligible flexural stiffness, then the beam response to the actuator force is linear,[5] and using Bernoulli-Euler beam theory, the following equation for the normalized beam surface strain is obtained:

$$\frac{\kappa t_b}{2\Lambda} = \frac{6\,(\bar{d}/T)}{\dfrac{E_b}{E_a} + \dfrac{2}{T^3} + 6\,\dfrac{\bar{d}^2}{T}} \tag{5}$$

Differentiating Eq. 5 with respect to $\bar{d}$ and setting it equal to zero leads to the following equation for the optimum offset distance:

$$\bar{d}_{\text{opt}} = \sqrt{\frac{E_b}{E_a}\frac{T}{6} + \frac{1}{3T^2}} \tag{6}$$

The physical explanation for the existence of an optimum offset distance is the same as given in the case of embedded actuators. Although the actuator force decreases with the increase in offset distance, the product of force and distance, i.e., the moment, increases up to the optimum offset distance; beyond that distance the decrease in force occurs at a faster rate than the increase in offset distance, and hence the moment is reduced. The beam surface strain normalized by the bonded actuator beam surface strain at the optimum offset distance is plotted in Fig. 4.

The idea of offset actuators is new, and compared with simple surface bonding it is certainly more demanding in terms of physical attachment, etc., but the payoff is enormous. As seen in Fig. 4, the moment applied to the substrate can be more than doubled for high thickness and modulus ratios. By filling the space between the actuator and the substrate surface with a honeycomb, and bonding the actuator to the honeycomb, the actuator is forced to deform with the same curvature as the substrate. This limits the ability of the actuator to cause deformation, compared with when the actuator is not forced to deform along with the structure with the same curvature (i.e., discretely attached with no filler in between). Thus, the actuator authority is decreased, but even then the enhancement compared with surface-bonded actuators is substantial.

Optimization in the Presence of External Loads

In the second part of the paper, the effect of external loads on the optimization parameters is considered. It will be shown that the parameters that result in optimum actuator performance without external loads are quite different when the actuator/structure is under the action of an opposing external load. The reason for this is quite simple: the actuator must now work against not only the substrate impedance but also the external loads, and for it to do this efficiently the optimum values must also change. Consider a beam with symmetric actuator patches (activated out of phase) working against an opposing external moment M, as shown in Fig. 5. The beam surface strain in such a situation is given by

$$\frac{\kappa t_b}{2} = \frac{1}{2\left[6 + \psi + \dfrac{8}{T^2} + \dfrac{12}{T}\right]}\left[12\left(1 + \frac{1}{T}\right)\Lambda - M^*\psi\right] \tag{7}$$

where

$$M^* = \frac{M t_b 10^4}{(EI)_b} \tag{8}$$

is the external moment suitably normalized by the beam flexural stiffness and thickness. Figure 6 shows the beam surface strain vs the thickness ratio for a free induced strain $\Lambda = 500\ \mu$ strain. As the external moment increases, the thickness ratio that results in maximum surface strain decreases. In addition, the optimum thickness ratio is now also a function of activation level.

Why this dependence on activation level? Faced with an opposing moment at a lower activation level, a higher actuator thickness performs better. This shift in the optimum thickness ratio because of an external moment is also easily explained by once again observing the through-the-thickness normal stress variation in Fig. 3. To get the stress on the top of the actuator surface to go to zero in the presence of an external load, clearly the thickness ratio must be changed. For example, to determine the external moment for which $t_b/t_a = 2$ is optimum, we can compute the moment (and the associated linear stress distribution) that, when superimposed on the stress distribution given in Fig. 3a, will result in zero stress on the top of the actuator.

As a second example, consider a more realistic problem of a beam clamped at both ends with a patch of symmetric actuators in

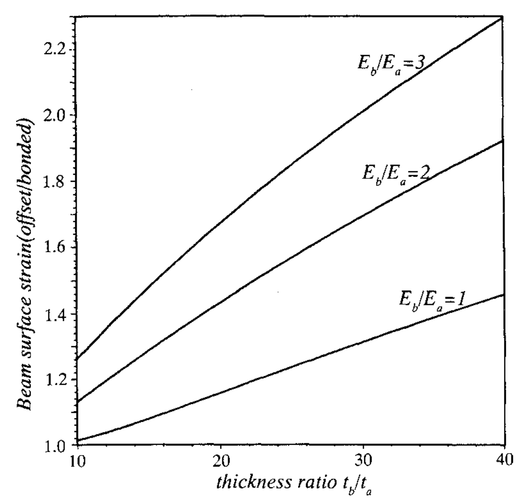

Fig. 4 Offset actuator surface strain/bonded actuator beam surface strain vs thickness ratio at optimum offset distance.

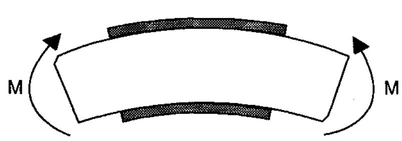

Fig. 5 Beam with symmetric patch of actuators working against an external moment.

the middle, as shown in Fig. 7. It should be immediately noticed that the clamped boundary conditions will have the same effect as an external load because they, like an external load, prevent the free motion of the beam actuator patch in the middle. In this case we choose to study the effect of another parameter on the performance of the actuator/substrate system. That parameter is the length of the actuator patch in the center of the beam; because of the clamped boundary conditions, the variation in the length of the patch has the same effect as increasing and decreasing an external load. The static response for this system is also contrasted with the conventional method of solving such problems where the actuator patch is replaced with a set of equivalent moments. The solution is easily obtained by dividing the beam into three parts and enforcing the boundary conditions and continuity of displacement, slope, shear, and moment at the junctions. There is a jump in moment at the boundary between the beam and the beam-actuator portion, and this boundary condition is written explicitly to point out the difference between the correct solution method and the solution obtained by using M_{eq}:

$$M_{\Lambda} - EI_1 w_{1_{,xx}} + EI_2 w_{2_{,xx}} = 0 \tag{9}$$

where

$$M_{\Lambda} = bE_a\left[t_a t_b\left(1 + \frac{1}{T}\right)\right]\Lambda \tag{10}$$

For convenience, the explicit expression for M_{eq} is also given:

$$M_{eq} = E_b b t_b^2 \frac{[1 + (1/T)]}{6 + \psi + (8/T^2) + (12/T)}\Lambda \tag{11}$$

If the equivalent moment M_{eq} is used to represent the action of the actuators; then M_{Λ} will be replaced by M_{eq} and EI_2 by EI_1 in Eq. (8), and thus the flexural stiffness will be constant throughout the length of the beam.

Figure 8 shows the beam displacement at the center δ normalized by the length l of the beam vs α for $t_b/t_a = 5$ and $E_b/E_a = 1$. The variable α represents the fraction of the beam length covered by the actuator patch. When $\alpha = 0$, there is no patch, and when $\alpha = 1$ the complete length of the beam is covered by the actuator patch. The error introduced in the static displacements by using M_{eq} is obvious. Table 1 lists the error for various values of α and two different thickness ratios. For a thickness ratio of 5, the error is 6.4% even for an actuator patch as small as 10% of the total length of the beam. For longer actuator patch lengths, the error as seen in Table 1 can be as high as 30–40%. Therefore, for such boundary conditions and longer actuator patches the use of M_{eq} to represent the action of the actuator on the structure can lead to a substantial error. It should be noted that if the boundary conditions were simply supported, there would be no difference in the two responses. Also of interest is the fact that a specific value of α maximizes displacements and beyond this value the displacements go down. The physical explanation for the displacements decreasing after a certain value of α is that as α increases, the ends of the actuator move closer to the fixed boundaries where the impedance of the structure is higher and the moment exerted by the actuator on the beam is unable to cause as much deflection.

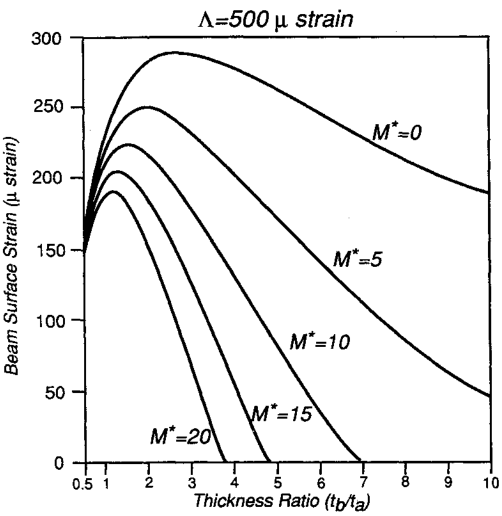

Fig. 6 Beam surface strain vs thickness ratio in the presence of an opposing external moment (Λ = 500). Note the change in optimum thickness ratio with external load.

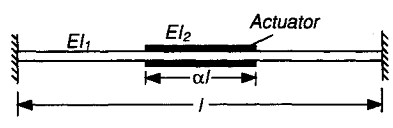

Fig. 7 Geometry of clamped-clamped beam with an actuator patch in the middle.

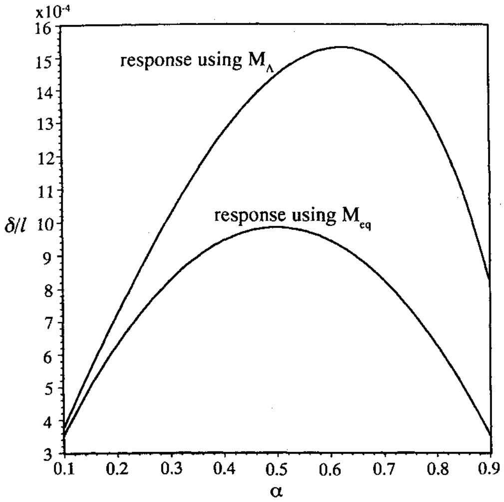

Fig. 8 Beam displacement at center (δ/l) vs α. Shown are the exact response, the response obtained by using M_{eq}, and the optimum length of the actuator patch.

Table 1 Error in static displacement due to M_{eq}

α, %	10	20	30	40	50	60
Error, %—$t_b/t_a = 5$	6.4	12.7	19.1	25.4	31.7	38.1
Error, %—$t_b/t_a = 10$	4.2	8.43	12.6	16.8	21.1	25.3

Effect of Inertia Loads on Performance and Optimization

In a vibration problem, the inertia loads are often dominant and in many instances are an order of magnitude higher than the static loads. A formulation that can account for all inertia loads (due to the structure as well as the actuator) and also correctly model the effect of the boundary conditions on the actuator performance can truly provide answers to the optimization question. In this section, first the limitations of the conventional method of dynamic analysis where the actuator is simply replaced by a set of moments or forces will be highlighted.

An actuator when bonded to a structure changes the host structure in two ways. First, it provides an additional stiffness in the region of bonding; second, it adds mass in that region. This local change in mass and stiffness even in a small region of the structure can alter some specific dynamic characteristics of the structure. The difficult part is that this modification of dynamic behavior is difficult to predict a priori. To demonstrate this, we once again consider a clamped-clamped beam with a pair of symmetrically bonded actuators located in the middle of the beam. The beam material is chosen to be aluminum (E = 10 × 10^6 psi). The beam is 0.06 in. thick and 12 in. long. For the actuators, the properties of G-1195 PZT actuator are used (E = 9.1 × 10^6 psi, ρ = 0.008566 slug/in.^{3}). The dynamic analysis was performed using the BEAM VI[6] program, with a structural damping of 0.1%; the actuators are driven by an out-of-phase electric field equivalent to a free induced strain of 100-μ strain. Figure 9 shows the frequency response at the center of the beam for the case where the actuator length is 20% of the beam length and the actuator thickness is one-fifth of the beam. The difference in the response using M_{eq} (as obtained from the Bernoulli-Euler model) and that obtained using M_Λ is considerable. The change in natural frequencies is also obvious. The natural frequencies of the beam with and without actuators and for different actuator lengths and thicknesses are summarized in Table 2.

It is interesting to note the shift in natural frequencies with the inclusion of actuator mass and stiffness. There is no consistent trend; some frequencies increase, some decrease, whereas others are relatively insensitive. There is actually a simple explanation for this behavior: the mass loading of the actuator dominates in the case of the first natural frequency causing the first natural frequency to drop; for the third and fifth natural frequencies, the stiffness is dominant and they increase. Because the actuator is located in the middle, the even frequencies corresponding to modes where the actuator is at a node are least affected. A similar trend is seen in the frequency response, where the actuator located at the node is unable to excite even modes. As seen in Table 2 the shift in natural frequencies can be considerable for longer and thicker actuators, and although there is a simple explanation for the shift in natural frequencies (increase or decrease), it is hard to predict the change without a complete analysis.

To illustrate the effect of the clamped boundary condition on the dynamic response, we compare the change in the first natural frequency of a pin-pin beam with and without actuators with the change in the natural frequency of the clamped-clamped beam in the example considered earlier. Both beams have geometrical properties that are identical to those of the earlier example. For a pin-pin beam with an actuator that is 20% of the length of the beam and one-fifth the beam thickness, the change in the first natural frequency with and without actuator is only 1.5 Hz, whereas for a clamped beam, as seen in Table 2, the change is about 8 Hz. Therefore, at low frequencies the noninclusion of actuator mass and stiffness can lead to a greater error in the case of clamped boundary condition.

Table 2 Shift in beam natural frequencies due to inclusion of actuator mass and stiffness

	Natural frequencies, Hz				
	First	Second	Third	Fourth	Fifth
Plain beam	85.229	235.130	460.950	761.974	1138.258
l_a/l_b = 0.1—t_b/t_a = 5	79.622	234.477	462.930	757.945	1156.237
t_b/t_a = 10	82.520	234.891	461.851	760.674	1144.379
l_a/l_b = 0.2—t_b/t_a = 5	77.192	231.417	482.206	755.360	1183.64
t_b/t_a = 10	80.835	233.699	468.371	759.869	1151.988

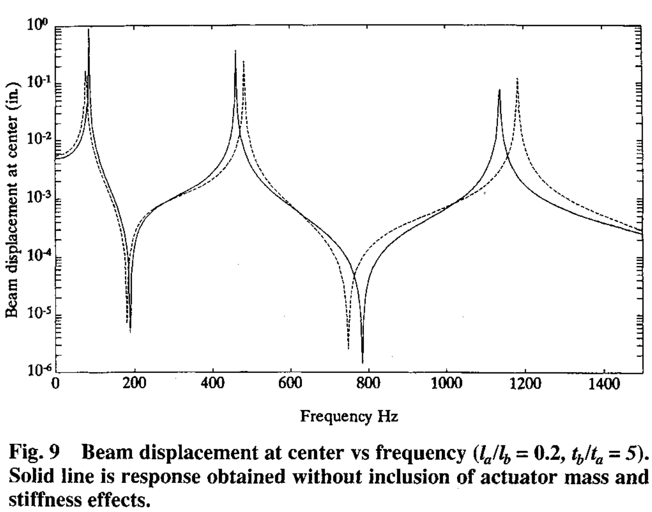

Fig. 9 Beam displacement at center vs frequency (l_a/l_b = 0.2, t_b/t_a = 5). Solid line is response obtained without inclusion of actuator mass and stiffness effects.

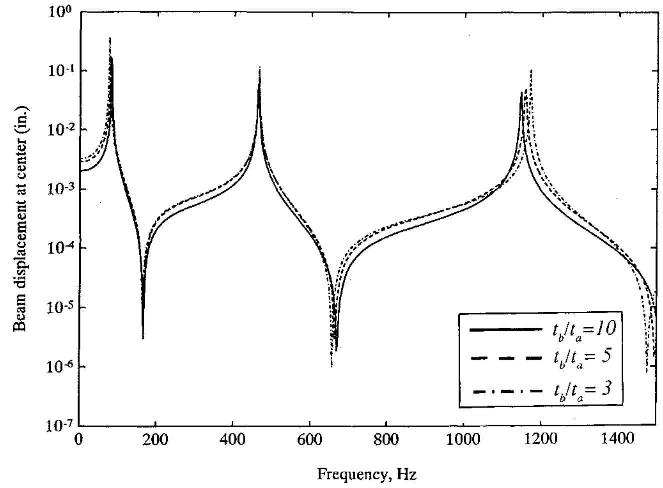

Fig. 10 Effect of thickness ratio on frequency response (l_a/l_b = 0.1).

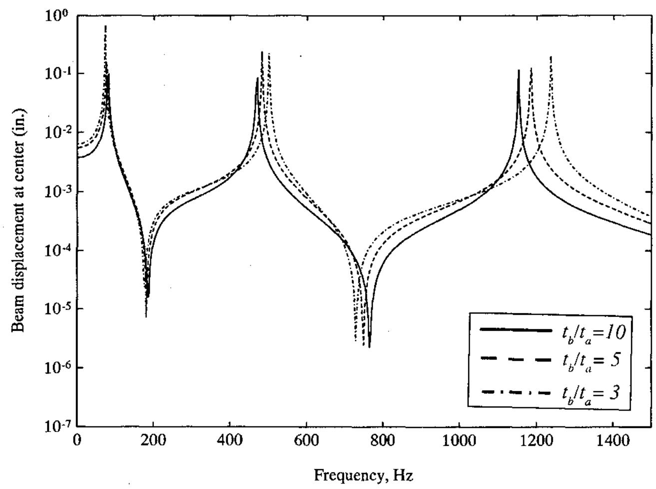

Fig. 11 Effect of thickness ratio on frequency ratio (l_a/l_b = 0.2).

Effect of Actuator Thickness on Frequency Response

In this section, the effect of the actuator thickness on the actuator performance measured in terms of the beam displacement at different locations is studied. For comparison three different beam-actuator thickness ratios—t_b/t_a = 3, 5, and 10—and two different actuator lengths—l_a/l_b = 0.1 and 0.2—are considered. The beam geometry, actuator material, and boundary conditions are the same as those in the preceding example. The actuator mass and stiffness are correctly accounted for by dividing the beam into three parts, and M_Λ (due to a 100-μ strain free induced strain) is used as the excitation moment in all cases. Figures 10 and 11 show the frequency response (displacement at beam center vs frequency) for l_a/l_b = 0.1 and 0.2, respectively. The difference in the static beam displacement for different beam-actuator thickness ratios is very distinct, but with increasing frequency the difference is not very distinct even at off-resonance frequencies. This is because the beam center is either a node or a point of maximum displacement and hence very sensitive to changes in frequency. The shift in natural frequencies due to the actuator mass and stiffness is again very obvious.

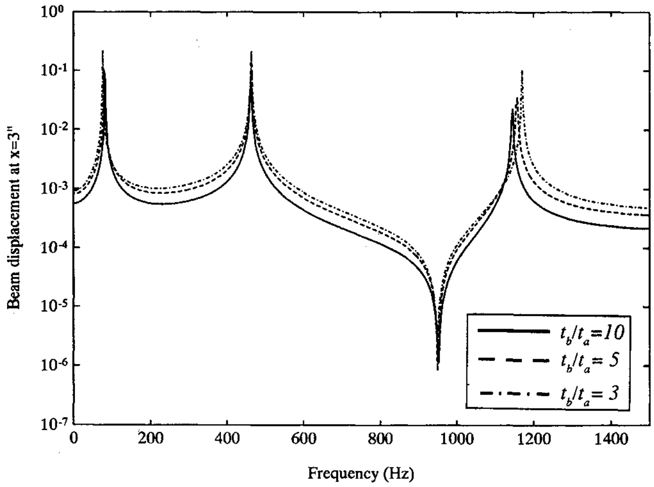

Fig. 12 Effect of thickness ratio on frequency response at 3 in. (l_a/l_b = 0.1).

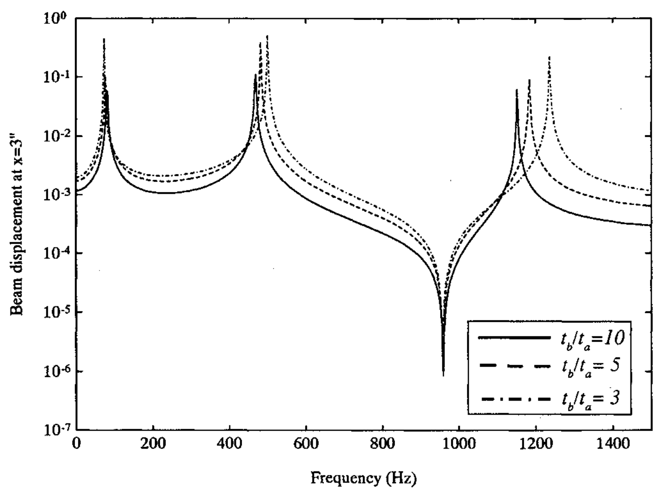

Fig. 13 Effect of thickness ratio on frequency response at 3 in. (l_a/l_b = 0.2).

To further study the effect of the actuator thickness, an arbitrary point on the beam that does not lie on a node was chosen. Figures 12 and 13 show the frequency response at a point that is one-quarter of the beam length from the clamped end. The performance of thicker actuators is very distinct at all off-resonance frequencies. The beam displacement at different off-resonance frequencies and the static displacement are compared in Table 3. For easy comparison the displacements are normalized by the displacement obtained for t_b/t_a = 10. This normalization scheme facilitates us in answering this question: does a thicker actuator provide the same degree of increase in displacements at all frequencies or not? The answer to this question is obviously no. With increasing frequency, thicker actuators are able to produce greater displacements compared with static and lower frequencies. For example, in Table 3, for l_a/l_b = 0.2 at 1500 Hz, an actuator one-third the thickness of the beam can produce four times as much displacement as an actuator that is one-tenth the beam thickness. Statically it can produce only 1.7 times the displacement. The trend is similar when the actuator is one-tenth the length of the beam (Fig. 12), although the difference is obviously smaller.

This analysis does not provide an answer to the actuator thickness optimization question, but it certainly provides some insight into the parameters involved, which are frequency, geometry, actuator length and location, boundary conditions, and, most importantly, the location on the beam where the vibration is to be minimized or maximized. In all probability, the actuator activation level, as in the static case, will also be an important variable in the actuator thickness optimization.

Table 3 Effect of actuator thickness on frequency response

		Normalized beam displacement at $l_b/4$, Hz			
l_a/l_b	t_b/t_a	Static	225	700	1500
0.1	3	1.623	1.834	1.963	2.223
	5	1.434	1.532	1.592	1.694
	10	1	1	1	1
0.2	3	1.697	1.982	2.746	3.974
	5	1.471	1.600	1.867	2.157
	10	1	1	1	1

Conclusions

In the first part of the paper, the actuator/substrate thickness ratio that maximizes the strain induced in the substrate (without external loads) is discussed. Three configurations are considered: 1) the actuator is embedded in the substrate; 2) the actuator is bonded to the surface of the substrate; and 3) the actuator is offset from the surface of the substrate. It is shown that embedding actuators just below the surface does not necessarily result in the maximum induced strain. A specific through-the-thickness normal stress distribution is shown to be the physical basis for the existence of an optimum thickness ratio for surface-bonded actuators. The equation for the optimum offset distance of discretely attached offset actuators is presented. For thicker substrates, optimally offset actuators can lead to a substantial increase in induced strain compared with surface-bonded and embedded actuators.

In the second part of the paper, the effect of external loads on the optimum thickness ratio is studied. The optimum thickness ratio changes with the external load, and it also changes with the actuator activation level. As the external load increases, the optimum thickness ratio decreases (i.e., the actuator thickness relative to the beam increases) to combat the higher load. Boundary conditions that prevent the free actuation of the structure are shown to have the same effect as external loads. As an example, a clamped-clamped beam with an actuator patch in the center is considered. Because of the clamped boundary conditions, the error due to the replacement of the actuator with a set of equivalent moments is found to be considerable. A specific length of the patch is shown to result in maximum induced strain. This length of the patch varies with the actuator-thickness ratio; for higher thickness ratios, it approaches half the beam length.

In a dynamic analysis, the inclusion of actuator mass and stiffness effects is necessary for the accurate prediction of the frequency response of the structure. Although thicker actuators perform better at higher frequencies, the present analysis is inconclusive regarding actuator thickness optimization in general.

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